

NONLINEAR CHARACTERISTICS OF JOINTS
AS ELEMENTS OF MULTI-BODY DYNAMIC SYSTEMS

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Introduction

As the connecting elements in Multi-Body structures, joints play a pivotal role in the overall dynamic response of these systems. Obviously, the linear stiffness of the joint strongly influences the system frequencies, but the joints are also likely to be the dominant sources of damping and nonlinearities, especially in aircraft and space structures. The general characteristics of such joints will be discussed. Then the state of the art in nonlinear joint characterization techniques will be surveyed. Finally, the impact that joints have on the overall response of structures will be evaluated.

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Although somewhat difficult to assess, the rough order of magnitude of various dissipative mechanisms is shown (based on critical damping equaling unity). In Earth-based structures, transmission losses probably dominate. But in aeronautical structures, dissipation in joints begins to become more important. In space, in the absence of transmission losses, joints dominate the passive dissipation mechanisms.

ORDER OF MAGNITUDE OF STRUCTURAL DISSIPATIVE MECHANISMS

	<u>EARTH</u>	<u>AERO</u>	<u>SPACE</u>
SUPPORT TRANSMISSION	10^{-1}	0	0
AEROACOUSTIC TRANSMISSION	10^{-2}	10^{-2}	0
MATERIAL DAMPING	10^{-3}	10^{-3}	10^{-3}
JOINT DAMPING	10^{-2}	10^{-2}	10^{-2}
ACTIVE CONTROL	10^{-1}	10^{-2}	10^{-2}

The potential nonlinear characteristics of a space structure are compared with the stiffness (normalized to unity). In the absence of yielding, material nonlinearities will be on the order of fractions of a percent. Geometric large deflection, at least in the flexible modes, is small. Therefore the strong nonlinearities of the joints are again likely to dominate.

ORDER OF MAGNITUDE OF SPACE STRUCTURAL NONLINEARITIES

MATERIAL STIFFNESS	10^{-3}
MATERIAL DAMPING	10^{-3}
GEOMETRIC LARGE DEFLECTION	10^{-2}
GEOMETRIC JOINT NONLINEARITY	10^{-1}

THEREFORE JOINTS ARE THE LARGEST SOURCE OF PASSIVE
DAMPING AND NONLINEARITY.

The overall characteristics of material damping, listed below, coupled with the fact that the material damping is likely to be one-half to one order of magnitude less than joint damping, tend to make this a relatively less critical area in modeling.

MATERIAL DAMPING CHARACTERISTICS

- DISTRIBUTED WITH STIFFNESS, THEREFORE MODAL DAMPING IS PROPORTIONAL, MODES ARE REAL AND UNCOUPLED
- ONLY WEAKLY NONLINEAR, THEREFORE APPROXIMATE, MODELS ARE SUFFICIENTLY ACCURATE
- HAS ORIGINS IN REASONABLY WELL-UNDERSTOOD MECHANISMS, E.G., THERMAL TRANSPORT, PLASTICITY
- IS DEPENDENT ON GLOBAL FREQUENCY, AMPLITUDE TEMPERATURE AND HUMIDITY ENVIRONMENT

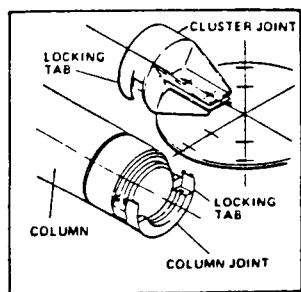
The corresponding characteristics of joints, their discrete locations, strongly nonlinear behavior, and somewhat obscure micro-mechanics, make this a more challenging area for modeling. Despite frequent attempts in the history of aerospace technological development, no unified analysis approach to this modeling has been developed.

JOINT CHARACTERISTICS

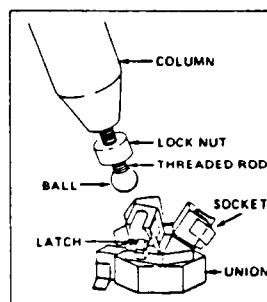
- NOT DISTRIBUTED, BUT OCCUR AT DISCRETE LOCATIONS, THEREFORE MODAL DAMPING IS NOT PROPORTIONAL, AND MODES ARE LINEARLY COUPLED AND COMPLEX
- STRONGLY NONLINEAR, THEREFORE MODES STIFFEN AND COUPLE NONLINEARLY
- HAS ORIGINS IN RELATIVELY POORLY UNDERSTOOD MECHANISMS, E.G., MICROSLIP FRICTION, IMPACTING

To gain some insight into this difficulty, it is useful to look at several proposed joint geometries for deployable space structures. Note that the geometries are all quite different, but all have several characteristics in common. There must be some amount of play in the joint to allow for assembly but some stiffening or locking mechanism to make the joint fixed when deployed. This combination of play and fixity leads to the impacting and nonlinear stiffness typical of such joints.

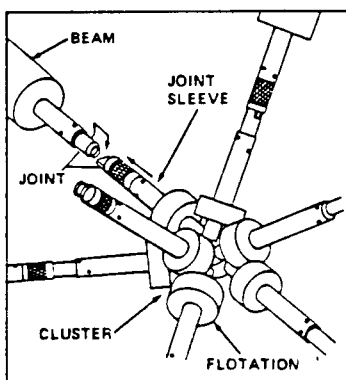
TYPICAL JOINT DESIGNS FOR DEPLOYED SPACE STRUCTURES



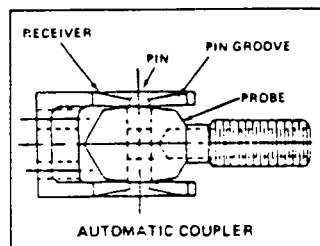
a. LaRC SNAP-JOINT UNION



b. RI BALL/SOCKET CONNECTOR



c. MIT CLUSTER SLIP-JOINT



d. VOUGHT QUICK-CONNECT COUPLER

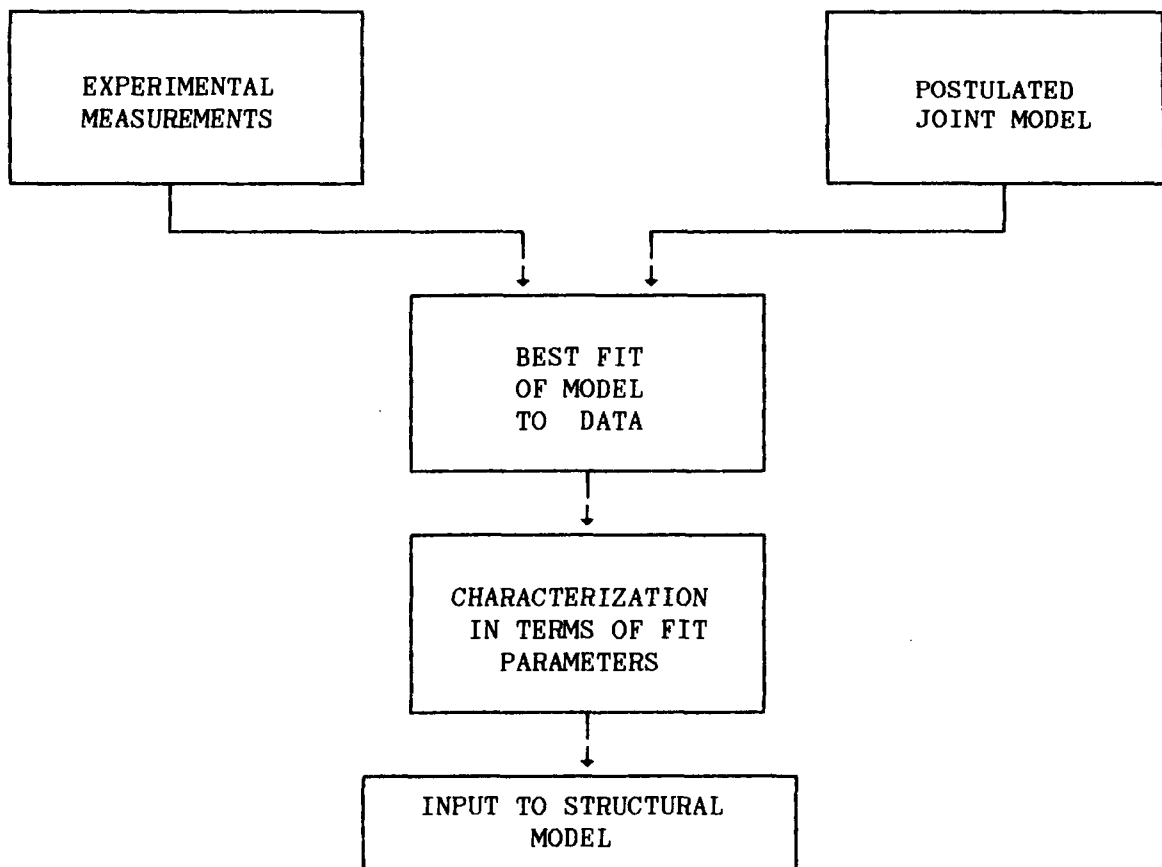
Not only do the properties of joints depend on the overall geometry of their design, but these properties depend on a number of details. The surface of the contacting elements can depend, for example, on the quality of machining, the load and wear history, and the duration on orbit. Even nominally identical joints can have a statistical variation due to manufacturing tolerance. Therefore, in realistic assemblies, direct calculation of properties is somewhat unproductive.

- JOINT PROPERTIES DEPEND ON VERY LOCAL DETAILS
 1. SURFACE FINISH, LUBRICATION, OUTGASSING
AND OXIDATION
 2. WEAR AND TRIBOLOGY
 3. PRECISION OF FIT AND ALIGNMENT
 4. PRELOAD AND INITIAL DEFORMATION
 5. LOCAL THERMAL DEFORMATIONS
- JOINTS OF IDENTICAL MATERIALS CAN HAVE VERY
DIFFERENT BEHAVIOR
- NOMINALLY IDENTICAL JOINTS MAY HAVE A STATISTICAL
VARIATION IN BEHAVIOR

THEREFORE THE DETAILED CALCULATION OF JOINT CHARACTERISTICS
FROM FIRST PRINCIPLES IS UNPRODUCTIVE.

A more common approach to characterization is a hybrid of simplified modeling and experimentation. A set of experiments is run, yielding some data on the force transmission of the joint. Concurrently, several postulated models of the joint are developed. Often this is somewhat interactive, i.e., after the data are evaluated, refined models are postulated. The force characteristics, or structural response of the postulated model, is then compared with the experimental data, and some fit of the model to data is performed. Based on this fit, the parameters of the model are available for use in the overall structural model.

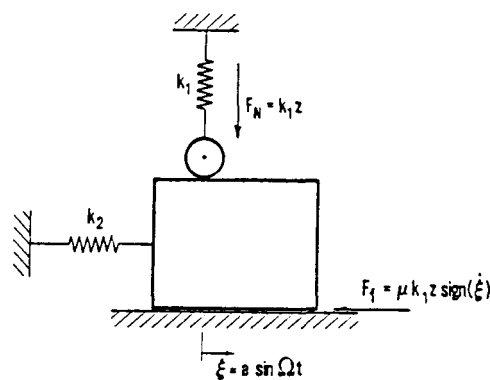
ALTERNATIVE TO DIRECT CALCULATION



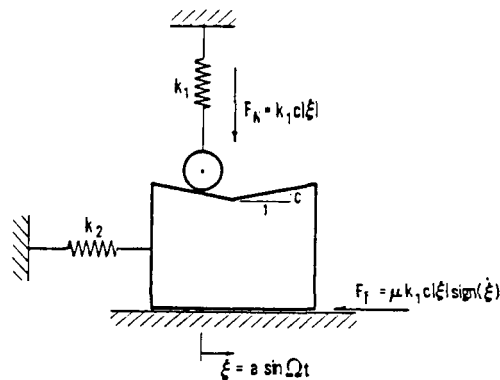
A number of different models of joint behavior can be postulated. Three of the common ones are shown below. The first is Coulomb friction, in which the normal force, and therefore the frictional drag force, remains constant. In displacement dependent friction, the normal and frictional drag forces vary with displacement. This model is probably more realistic for jointed trusses in the absence of thermal and gravity loads. When the deadband closes, impacting occurs, and a sharp jump in damping and stiffness (not shown) occurs.

EXAMPLE POSTULATED PIECEWISE LINEAR MODELS

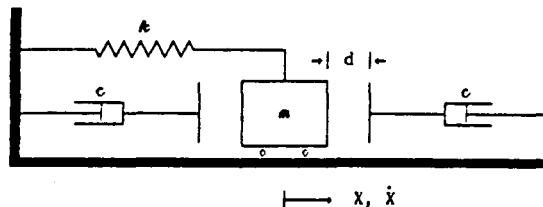
COULOMB FRICTION



DISPLACEMENT DEPENDENT FRICTION [1]



IMPACTING



The principal characteristics of four procedures for identification of nonlinear elements are shown. The first two are extensions of techniques developed for linear systems and are more easily extendable to multi-dof-models. However, they are probably only appropriate for weak nonlinearities. The latter two are currently limited to single-dof systems, but can handle stronger nonlinearities. A more detailed explanation of each will follow.

EXPERIMENTAL IDENTIFICATION PROCEDURES

<u>PROCEDURE</u>	<u>MEASUREMENT</u>	<u>DOMAIN OF FIT</u>	<u>NONLINEARITY</u>	<u>DoF</u>
FREQUENCY DOMAIN				
MODAL IDENTIFICATION [2] (EWINS, IMP.COL.)	$F, \ddot{x} \text{ vs } \omega$	FREQUENCY	WEAK	SEVERAL
TRANSIENT TIME				
DOMAIN [3] (HORTA, JUANG, LARC)	$F, \ddot{x} \text{ vs } t$	TIME	WEAK	SEVERAL?
CLASSICAL FORCE-				
DISPLACEMENT [4] (SONI, UDRI)	$F \text{ vs } x$	PARTIAL STATE SPACE	STRONG	ONE
FORCE-STATE [5] (CRAWLEY, MIT)	$F \text{ vs } x, \dot{x}$	STATE SPACE	STRONG	ONE

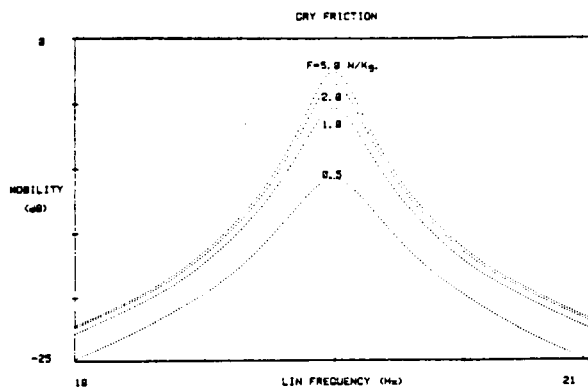
The extended frequency domain modal identification procedure was developed simply to uncover the presence of nonlinearities in modal test data. Therefore, the output is limited to indications of the presence, strength and type of nonlinearity. It is best used as a diagnostic tool in checking the consistency of test data.

EXTENDED FREQUENCY DOMAIN MODAL PROCEDURE [2]

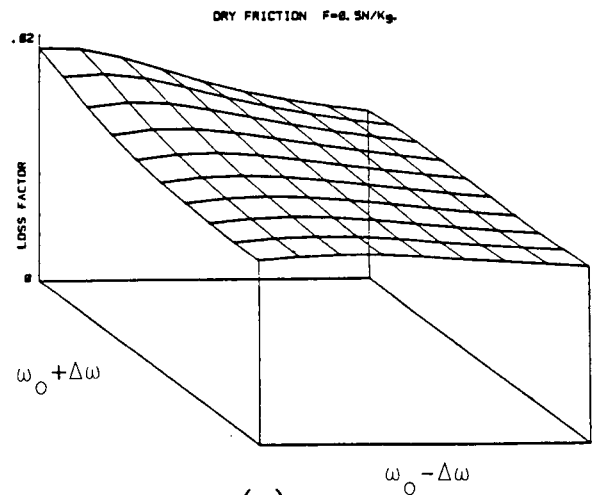
- POSTULATE NONLINEARITY AND CALCULATE LOSS FACTOR USING NYQUIST PLANE RESPONSE
- MEASURE RESPONSE AT RESONANCE AND CALCULATE LOSS FACTOR
- IF LOSS FACTOR IS INCONSISTENT (I.E., NOT CONSTANT), CHOOSE POSTULATED NONLINEARITY WHICH BEST FITS OBSERVED BEHAVIOR
- OUTPUT - APPROXIMATE INDICATION OF TYPE AND DEGREE OF NONLINEARITY IN MODAL RESONANCE.

The technique is a direct extension of the procedure for extracting frequency and loss factor parameters from transfer functions, as presented in the Nyquist plane (b). A simple, single-dof response appears as a perfect circle in this representation, tangent to the real axis at the origin. Any deviation from this circle is due to a nonlinearity (or presence of multiple poles). The loss factor (damping ratio) can be calculated by choosing pairs of points about ω_0 , forming a matrix of computed values. Figure (c) is a graphical representation of loss factor calculated on this matrix. For a linear system, this surface would be flat. The shape shown is typical of a system with Coulomb friction.

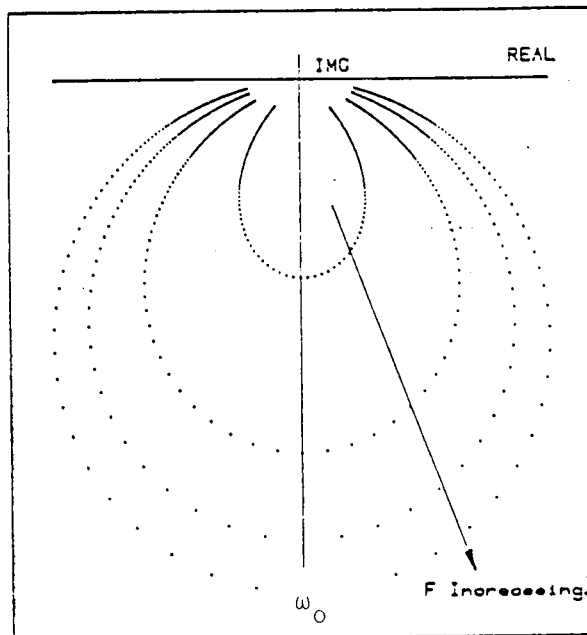
CALCULATED FREQUENCY RESPONSE (A) AND NYQUIST REPRESENTATION (B) FOR A SDOF SYSTEM WITH COULOMB FRICTION. THE LOSS FACTOR (C) IS INFERRED FOR A RANGE OF FREQUENCY SPREAD ABOUT THE RESONANCE.



(A)



(C)



FREQUENCY RANGE : 19 Hz TO 21 Hz.

(B)

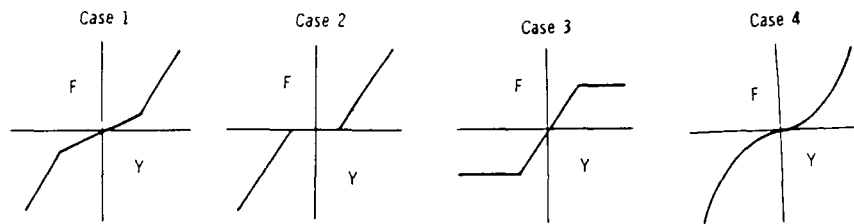
Likewise, the existing time domain techniques are extensions of techniques developed for linear systems. These techniques generally examine the transient free response to extract system mode shapes and frequencies. Weak nonlinearities appear as a frequency with a number of higher harmonics. Each type of nonlinearity has such a signature.

EXTENDED EIGENSYSTEM REALIZATION ALGORITHM [3]

- POSTULATE NONLINEARITY AND CALCULATE FOURIER CONTENT OF TRANSIENT FREE RESPONSE
- MEASURE FREE RESPONSE AND IDENTIFY FOURIER CONTENT WITH ERA
- COMPARE MEASURED HIGHER HARMONIC CONTENT OF OF MODAL RESPONSE WITH SIGNATURES OF POSTULATED NONLINEARITIES
- OUTPUT - APPROXIMATE INDICATION OF TYPE AND DEGREE OF NONLINEARITY IN RESPONSE.

Four example cases are shown, all typical of a stiffening or softening spring. The Fourier components of the free response of such a spring in a spring mass system were calculated. The calculated response was also fed as simulated data to the ERA program and the harmonics of the response calculated. Good capability to reconstruct the signature of a known non-linearity is shown. However, the recognition of the signature of an unknown nonlinearity is still under development.

FOUR GENERIC NONLINEAR JOINTS (A) AND
THE FOURIER CONTENT OF THEIR TRANSIENT
DECAY FROM ANALYSIS AND ERA IDENTIFICATION
OF COMPUTED RESPONSE (B).



(A)

CASE NO.	FREQUENCY Hz.		COMPONENT AMP.	
	ERA	ANALYSIS	ERA	ANALYSIS
1	0.135	0.134	0.986	0.987
	0.404	0.403	0.015	0.015
	0.673	0.672	-0.002	-0.002
2	0.125	0.125	0.979	0.979
	0.375	0.375	0.023	0.023
	0.625	0.625	-0.003	-0.003
3	0.096	0.096	1.030	1.030
	0.289	0.289	-0.035	-0.035
	0.482	0.482	0.006	0.006
4	0.456	0.456	0.961	0.961
	1.369	1.369	0.038	0.038
	2.281	2.281	0.001	0.001

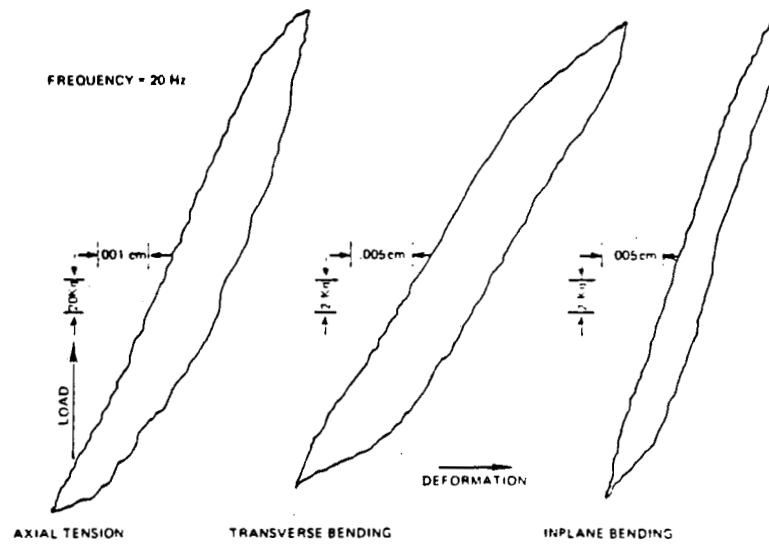
(B)

The classical approach to the problem is, of course, to simply put a joint in a testing machine and develop force-stroke data, as shown on the next page. From such data secant modulus and average loss factor can be calculated. The limitation is that such properties are already smeared, or averaged over the stroke, and no tangent or point properties can be determined. Further, the dependence of the force on the rate of change of stroke is lost.

QUASI-STEADY FORCE DEFLECTION PROCEDURE [4]

- POSTULATE NONLINEARITY AND CALCULATE ITS F VS X BEHAVIOR
- MEASURE F VS X BEHAVIOR FOR ONE X_{MAX} AND ω , AND CALCULATE EFFECTIVE STIFFNESS AND LOSS FACTOR
- REPEAT AT DIFFERENT AMPLITUDES AND FREQUENCIES
- OUTPUT - EFFECTIVE STIFFNESS AND LOSS AS A FUNCTION OF FREQUENCY AND AMPLITUDE.

TYPICAL FORCE-STROKE CYCLES OF THE
THREE STRESS RESULTANTS OF A JOINT
AT ONE LOAD AND FREQUENCY.



The force-state mapping procedure is designed specifically to identify strong nonlinearities in joints and addresses the two limitations of the classic Force-Stroke measurement. The dependence of transmitted force on both displacement and velocity is explicitly determined, and local or tangent values produced. At the current time the procedure is limited to single-dof systems.

FORCE-STATE MAPPING PROCEDURE [5]

- POSTULATE NONLINEARITY AND CALCULATE
F VS x, \dot{x} BEHAVIOR
- MEASURE F VS x, \dot{x} OVER EXPECTED RANGE
- FIT POSTULATED SURFACES TO DATA IN
FORCE-STATE SPACE
- OUTPUT:
 1. RAW DATA FOR DATA LOOK-UP
 2. GLOBAL FIT PARAMETERS FOR ANALYTIC
REPRESENTATION
 3. LOCAL EQUIVALENT SECANT MODULI
FOR LINEARIZED REPRESENTATION

The force-state maps of three simple systems are shown. A spring (a) produces a plane with a slope against x , but no change in \dot{x} . A linear viscous damper would produce a plane with a slope against \dot{x} , and no change in x . Thus any linear element (i.e., spring and damper) will have a map which is a flat plane in force-state space. Any deviation from a plane is indicative of a nonlinearity.

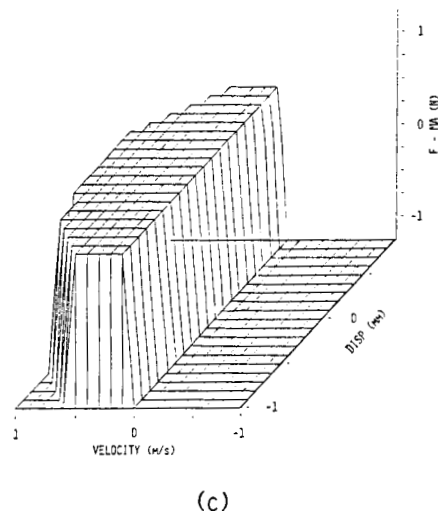
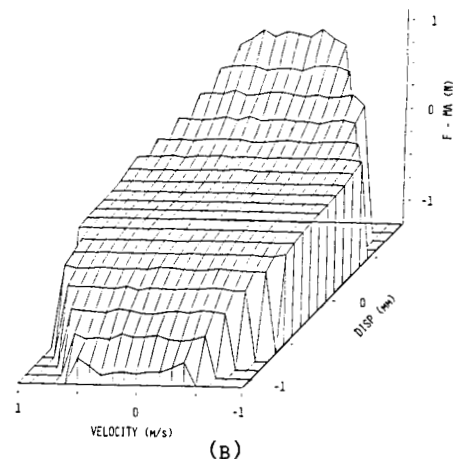
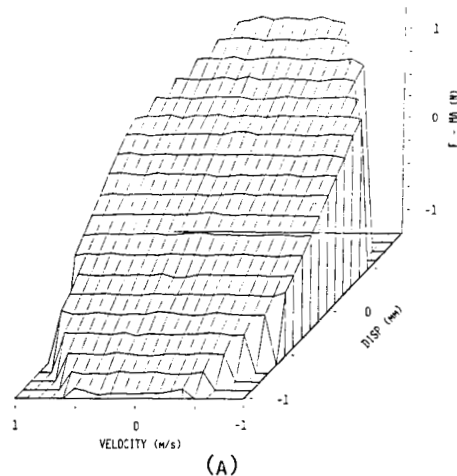
Two common nonlinearities are shown in (b) and (c). The cubic spring nature is clear in fig. (b). Figure (c) shows the map of Coulomb friction, which is independent of x , and takes on the sign of the velocity.

FORCE-STATE MAPS OF:

A) LINEAR SPRING

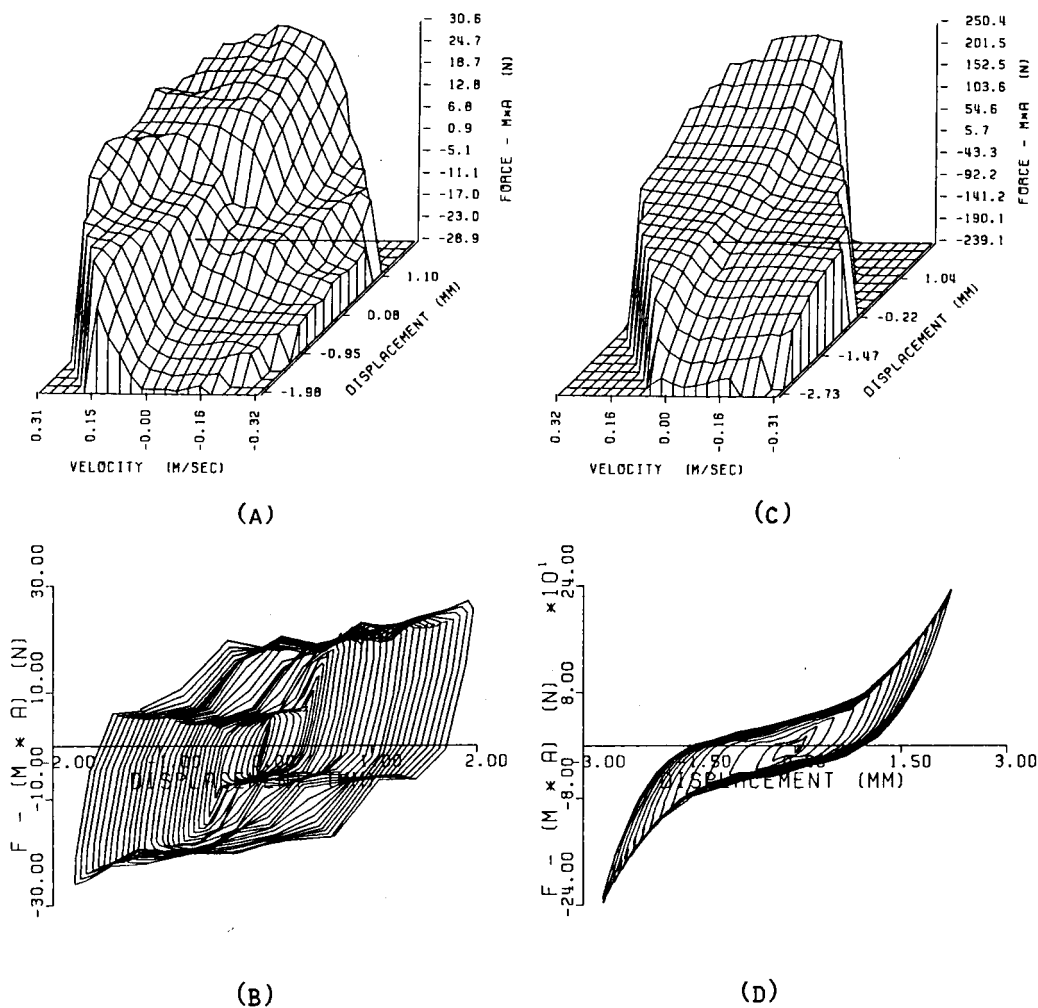
B) CUBIC SPRING

C) COULOMB FRICTION



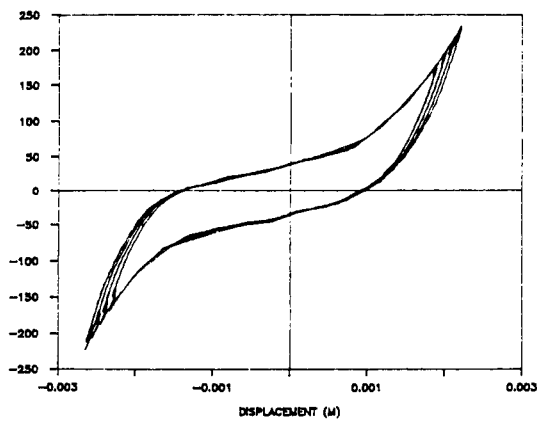
The force-state maps of a real joint are shown below. The joint is of a quick-disconnect-pin and clevis type, similar to the Vought connector shown earlier. The figures on the left (a and b) show the characteristic without a stiffening sleeve. Note the step at \dot{x} equals zero, indicative of friction. With the addition of a sleeve, the joint becomes stiffer (note the change in vertical scale), and the cubic stiffness of the sleeve begins to dominate. A strong dissipative nature is still obvious from the hysteresis loops in fig. (d).

THE FORCE-STATE AND FORCE-DISPLACEMENT
CURVES OF A CLEVIS-PIN-TYPE CONNECTOR
WITHOUT (A AND B) AND WITH (C AND D)
A REINFORCING SLEEVE.

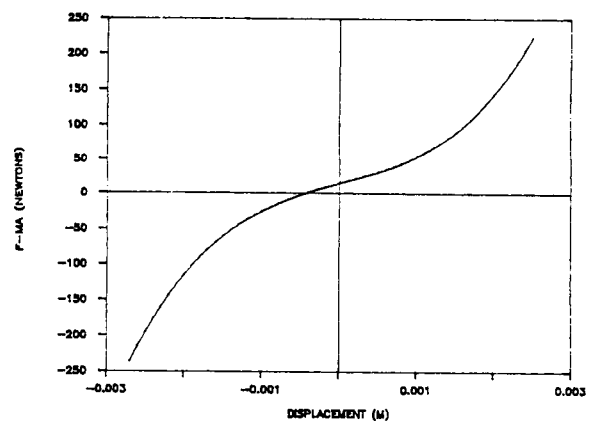


In an effort to fit a postulated model to the data on the previous page (i.e., the joint with sleeve), three successively refined models were used. In fig. (b), a cubic, plus linear, spring term approximately matches the shape but, of course, has no dissipative nature. The addition of friction introduces the classic hysterectic step. Finally, with the introduction of linear damping, the measured data are closely reproduced.

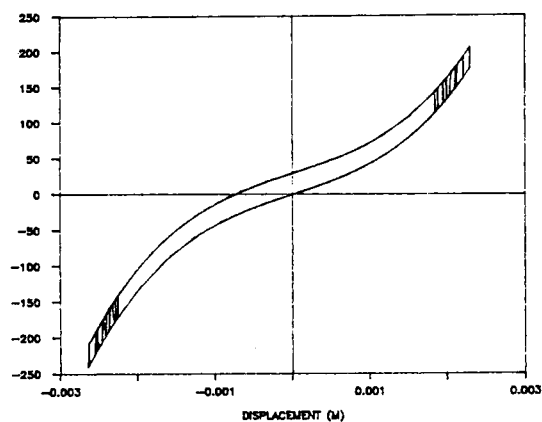
SUCCESSIVE APPROXIMATIONS OF ACTUAL DATA (A) BY A CUBIC SPRING (B),
CUBIC SPRING, PLUS FRICTION (C) AND CUBIC SPRING, FRICTION AND LINEAR
DAMPING (D).



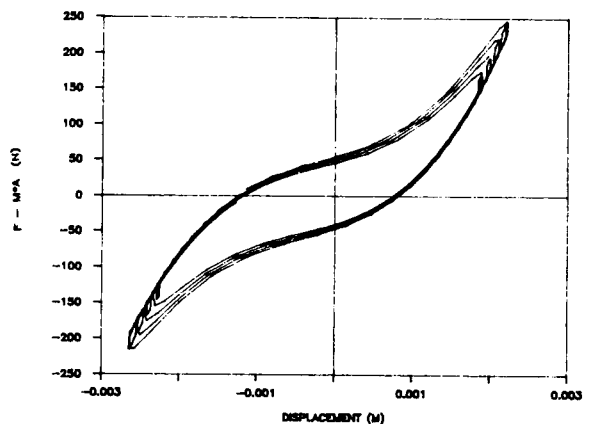
(A)



(B)



(C)



(D)

The requirements for efficient computation place several requirements on the identification scheme. It is highly desirable to have available the force-state information for direct pseudo-force computation.

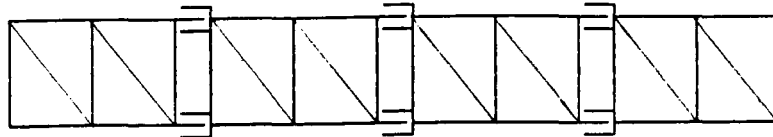
COMPUTATIONAL CONSIDERATIONS [6]

- THREE COMPUTATIONAL APPROACHES TO INCLUDING THE JOINT NONLINEARITY CAN BE CONSIDERED
 1. HOMOGENEOUS NONLINEARITY, EXPLICIT OPERATOR
 2. HOMOGENEOUS NONLINEARITY, IMPLICIT OPERATOR
 3. PSEUDO-FORCE REPRESENTATION [7]
- IN ALL THREE, BUT ESPECIALLY IN THE PSEUDO-FORCE METHOD, IT IS NECESSARY TO HAVE THE JOINT CHARACTERISTIC IN TERMS OF JOINT STATE VARIABLES.
- IF ONLY AVERAGE, OR SECANT PROPERTIES ARE KNOWN, THEN CONSIDERABLE ITERATION IS REQUIRED, AND TRANSIENT ANALYSIS MAY NOT BE ACCURATE.

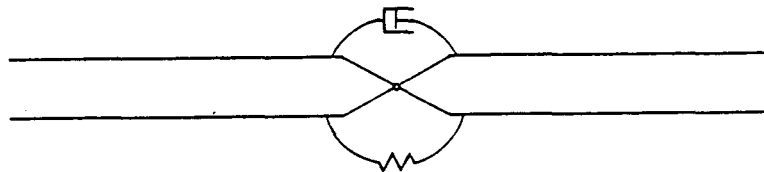
As an example problem, a four-bay truss, connected by joints, is modeled as a four segment beam, pinned in translation. In rotation it is constrained by a linear spring and damper.

EFFECTS OF JOINTS ON MODAL PROPERTIES

MODEL A CONNECTED 2-D TRUSS

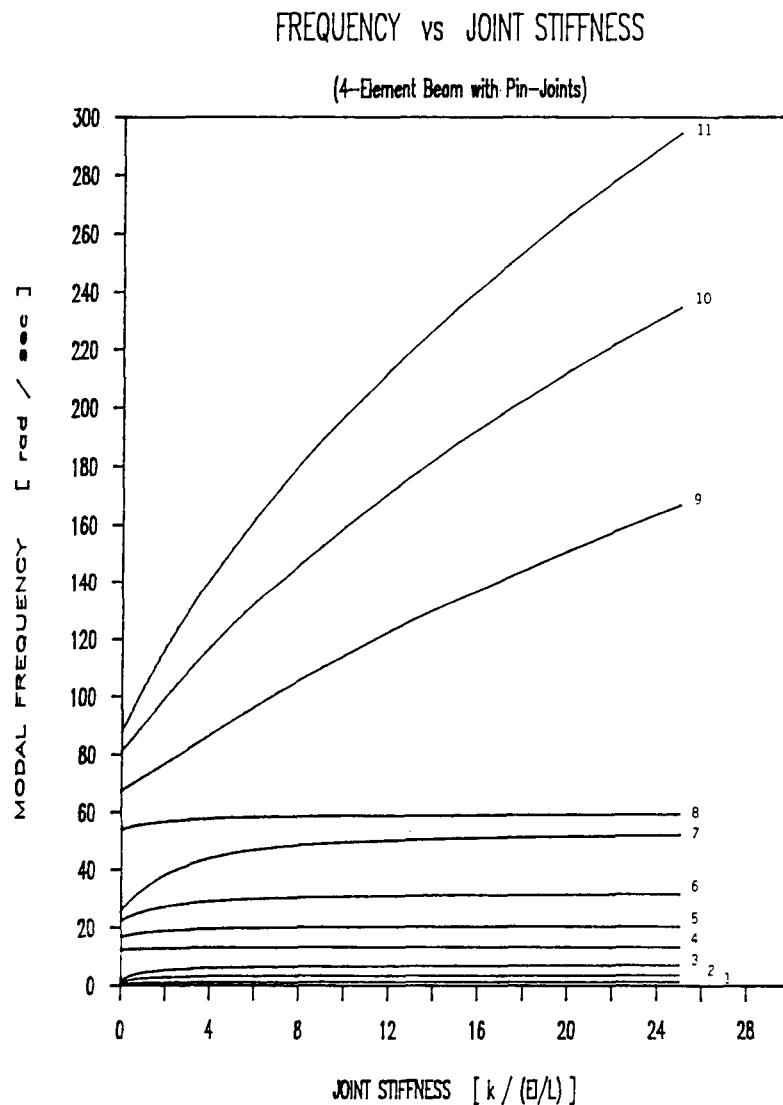


AS A PINNED BEAM OF 4 ELEMENTS WITH
ROTARY SPRINGS AND DAMPERS.



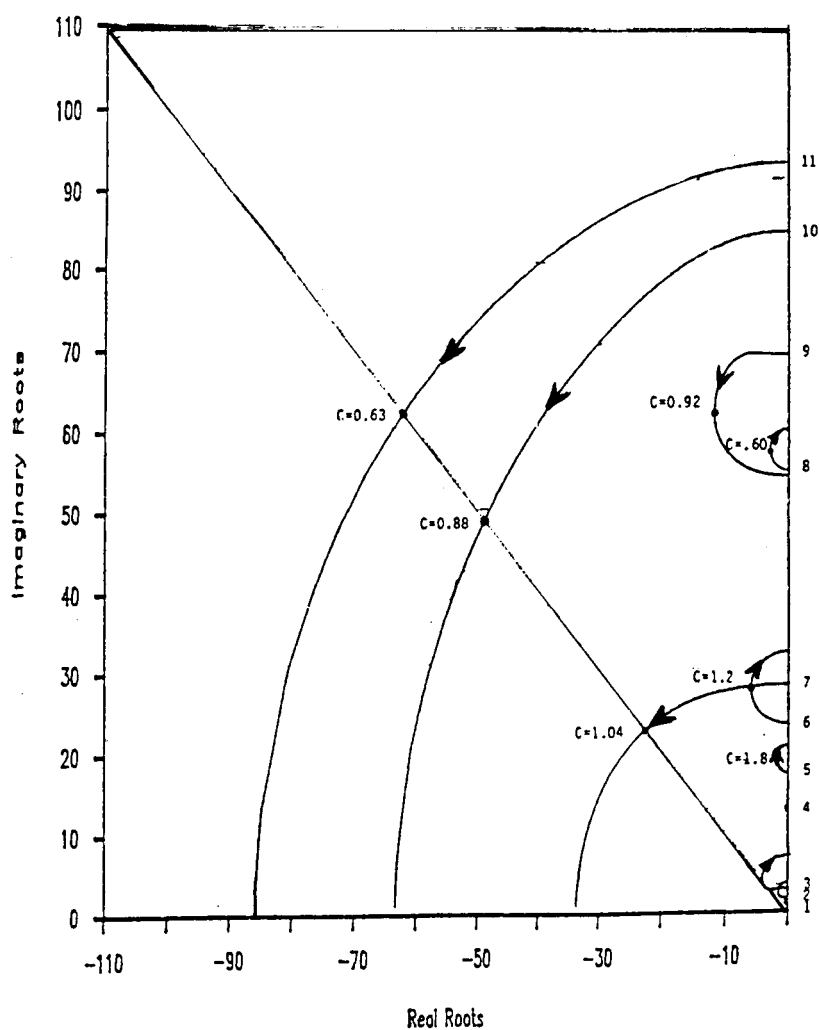
When the natural frequencies of the system are plotted versus nondimensional joint stiffness, their trends are apparent. Of course, all modes stiffen as k is increased. Some modes, such as #4, are only slightly affected, while others, such as #7, are strongly affected. The lowest eight modes are asymptotic to a constant frequency, while the highest three continue to rise as k increases.

STIFFENING EFFECTS OF JOINTS AS A FUNCTION OF K_{JOINT}



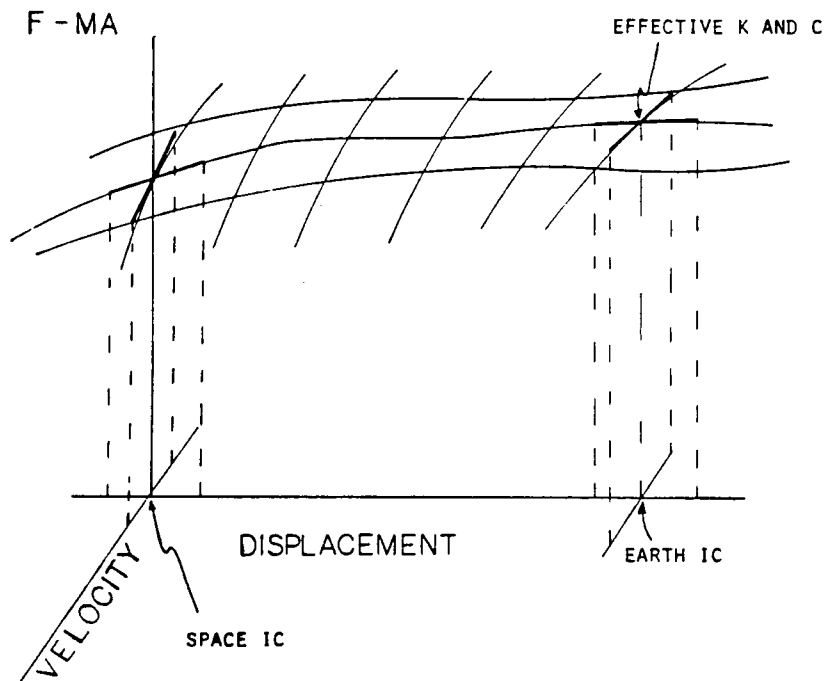
The addition of linear joint damping has some surprising results. Note that in only three modes, 7, 10 and 11, is the damping roughly proportional, i.e., the pole is driven to the real axis. In most modes, the root damps, then asymptotically stiffens and loses damping. In one mode, #9, the frequency drops.

LOCUS OF ROOTS FOR INCREASING LINEAR JOINT DAMPING, FOR $K_{JOINT} = 0.3 EI/l$



Finally, this figure shows an interesting application of the force-state map to Earth testing of space structures. Suppose a structure was suspended in one gravity in such a way that the gravity load caused a steady deflection. The small displacement vibration would then take place about this "Earth IC," and would have the effective stiffness and damping shown. In space, in the absence of gravity loads, there would be no steady deflection and the effective K and C would be about a "Space IC," as shown. For a generally nonlinear joint, these properties could be completely different from those of the Earth test, leading to differences in dynamic behavior on orbit when compared to those measured on Earth.

USE OF THE FORCE-STATE MAP TO DETERMINE
THE EFFECTIVE STIFFNESS AND DAMPING IN A
JOINTED STRUCTURE, AS WOULD BE MEASURED
ON EARTH AND IN SPACE.



SUMMARY

- DETAILED MODELING OF MICROMECHANICS
OF THE JOINT NOT PRODUCTIVE
- DEVELOPMENT OF SIMPLE GENERIC MODELS
USEFUL
- IMPROVED NONLINEAR IDENTIFICATION
NECESSARY.

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